

## Improved uncertainty propagation method of dynamic model for Mars entry spacecraft

FU Hui-min, XIAO Qiang, WU Yun-zhang

(Research Center of Small Sample Technology,  
Beijing University of Aeronautics and Astronautics, Beijing 100191, China)

**Abstract:** In order to research the uncertainty propagation laws of Mars entry dynamic equations for the Mars entry phase, an improved method was presented for analyzing the effect of the initial state uncertainties and uncertainty factor on the system state in the state trajectories. When applying the method to the entry phase of one of the NASA Mars exploration missions, the simulation results agreed well with the Monte Carlo method, especially the flight path angle simulation at least reaching 92%. It is found that the improved method can not only predict the uncertainty propagation laws with high accuracy of at least 92% in flight path angle simulation and large application scope from  $-0.1$  degree to  $0.1$  degree, compared with local linearization method, but also save several hours relative to Monte Carlo method.

**Key words:** uncertainty propagation laws; Mars entry; dynamic equations; improved method; high accuracy

**CLC number:** V41

**Document code:** A

Mars exploration missions play an important role in the future deep space exploration activities, and the number of countries and international originations, which have begun to focus planetary exploration activities on Mars, is increasing as time going. However, engineering interplanetary journeys are very complicated, so the exploration of Mars has experienced a high failure rate, especially in earlier attempts. Roughly two-thirds of all spacecraft destined for Mars failed before completing their missions, and there are some that failed before their observations could begin. One of the significant engineering challenges for delivering spacecraft from Mars orbit to surface involves the EDL (entry, descent and landing) phase of the mission. The Mars entry phase is defined as the period of the mission from the atmosphere interface to parachute deployment, which is considered as the critically important phase and decides the success

or not of the entire Mars exploration mission<sup>[1-2]</sup>. As experienced with the past successful landing missions<sup>[3]</sup>, it is widely assumed that the Mars entry interface begins at 125 km altitude. To date, all Mars spacecrafts continue to rely on the entry technologies developed for the Viking missions. However, Viking-like spacecraft leads to larger landing error ellipse in order of several hundred kilometers and cannot meet the requirements of future deep space exploration activities<sup>[4-6]</sup>.

Future Mars missions, such as Mars sample return and Mars base, need to obtain the pinpoint landing Mars within tens of meters to 1 km of a preselected target site<sup>[7]</sup>. Given the analysis before<sup>[4]</sup>, some factors, which contain the initial spacecraft states uncertainties, Mars atmosphere density, and lift-to-drag ratio, are the main reasons which will significantly lower the landing precision. As a result of that, to research the

**Received:** 2013-02-01

**Foundation item:** National Basic Research Program of China (2012CB720000)

**E-mail:** fuhuimin@263.net

landing errors caused by the uncertainties through some optimal methods<sup>[8]</sup> is necessary.

As to the uncertainty propagation problems for nonlinear dynamical system, there are some traditional methods, which contain MC (Monte Carlo) method<sup>[9]</sup>, MCMC (Markov chain Monte Carlo) method<sup>[10]</sup>, LL (local linearization) method<sup>[11-12]</sup>, UKF (unscented Kalman filter) method<sup>[13]</sup>, CTC (cell to cell) mapping<sup>[14]</sup>, PC (polynomial chaos) based method<sup>[15]</sup> and Fokker Planck Kolmogorov framework<sup>[16]</sup>, to evaluate the results in order to obtain the laws of uncertainty propagation.

Among the methods presented above, an improved method (IM method for short) based on local linearization method is proposed so that it meets the requirements of accuracy and efficient in deep space exploration. Assuming that the initial uncertainties are Gaussian distributed in Mars entry phase, the applicability of this method is necessary to research. So the aim of this paper is to obtain the laws of uncertainty propagation and the application scope of this method. This paper is structured as follows: section 1 describes the Mars entry problem. And the uncertainty propagation method for Mars entry problem, which is based on local linearization method, is inferred in section 2. Given the approach is applied to the entry scenario of a Mars exploration mission, the simulation results are presented and analyzed in section 3. The last section 4 contains the conclusions of this paper.

## 1 Mars entry problem

### 1.1 Mars entry dynamic equations

Typically, the Mars entry phase starts when the spacecraft starts at the Mars atmosphere interface and ends with the parachute deployment. As the Mars entry phase only lasts a short time, and the decrease of height is much less than the radius of Mars, a uniform gravitational field is used here and the Mars' rotation is neglected. It is also assumed that the entry spacecraft flies at a constant angle of attack and the controlled guidance of spacecraft is achieved

only through modulating the bank angle of entry spacecraft. Based on the Mars centered Mars-fixed coordinate system, the dynamic equations of Mars entry spacecraft are given by

$$\begin{aligned}\dot{r} &= v \sin \gamma \\ \dot{\theta} &= \frac{v \cos \gamma \sin \phi}{r \cos \lambda} \\ \dot{\lambda} &= \frac{v \cos \gamma \cos \phi}{r} \\ \dot{v} &= -d - g \sin \gamma \\ \dot{\gamma} &= \frac{l \cos \phi}{v} - \left( \frac{g}{v} - \frac{v}{r} \right) \cos \gamma \\ \dot{\psi} &= \frac{v \sin \phi \cos \gamma \tan \lambda}{r} + \frac{l \sin \phi}{v \cos \gamma}\end{aligned}\quad (1)$$

Where  $r$  is the distance from the center of Mars to the center of mass of entry spacecraft;  $\theta$  is the longitude and  $\lambda$  is the latitude;  $v$  is the velocity of entry spacecraft;  $\gamma$  is the FPA (flight path angle). The azimuth angle  $\phi$  is defined as a clockwise rotation angle from north, and  $\psi$  is the bank angle defined as the angle about the velocity vector from the local vertical plane to the lift vector.

The variables of drag and lift are presented by  $d$  and  $l$  as followed:

$$\begin{aligned}d &= 0.5 \rho v^2 / B \\ l &= d \times L / D\end{aligned}\quad (2)$$

Where  $B$  presents the ballistic coefficient and  $L/D$  denotes the lift-to-drag ratio.

Then, the gravitational acceleration  $g$  uses the Newton formulation which gives enough accuracy (roughly 99%).

$$g = \frac{\mu}{r^2}\quad (3)$$

Where  $\mu$  is the gravitational constant of Mars ( $\mu = 42828.29 \times 10^9 \text{ m}^3/\text{s}^2$ ).

Finally, an exponential Mars atmosphere density model is assumed as followed:

$$\rho = \rho_0 \exp \left[ - \left( \frac{r - r_0}{h_s} \right) \right]\quad (4)$$

Where  $\rho_0$  is the reference density on the surface of the Mars ( $\rho_0 = 2 \times 10^{-4} \text{ kg/m}^3$ );  $r_0$  is the reference radial position ( $r_0 = 3437.2 \text{ km}$ ), and  $h_s$  is the constant scale height ( $h_s = 7500 \text{ m}$ ).

## 1.2 Uncertainty factor of atmosphere density

In the Mars entry phase, there are lots of uncertainties in the real navigation system, such as initial entry state uncertainties, atmosphere model uncertainty and so on. But atmosphere model uncertainty, which is one of the main factors, affects navigation accuracy. So there is a demand to research the atmosphere model uncertainty.

And percentage is traditionally given as the unit of atmosphere model uncertainty. Furthermore, the equation for atmosphere model uncertainty is expressed as followed:

$$\rho_0(\Delta) = \rho_0^* + q \times \Delta \times \rho_0^* \quad (5)$$

Where  $\rho_0^*$  is the nominal density; the standard deviation of atmosphere model uncertainty is  $q$ , and  $\Delta$  complies with Gaussian distribution. Moreover, its mean and standard deviation are zero and one respectively.

## 2 Uncertainty propagation on improved method

In fact, the system described by Eq. (1) – Eq. (5) can be simply rewritten as

$$\dot{\mathbf{x}} = \mathbf{h}(t, \mathbf{x}, p, \phi) \quad (6)$$

Where  $p$  is the modified uncertainty factor (see in the next equation);  $\mathbf{x}$  is the state vector of system and the control variable is  $\phi$ .

Because  $p$  is a random constant parameter, Eq. (5) can be rewritten as

$$\begin{aligned} \dot{p} &= 0 \\ \rho_0(\Delta) &= \rho_0^* + p \times \rho_0^* \end{aligned} \quad (7)$$

Where  $E[p(t_0)] = 0$ ,  $E[p^2(t_0)] = q^2$ , and

$p(t_0)$ , which is Gaussian distributed, denotes the initial atmosphere model error.

So, let the state vector be  $\mathbf{X} = [\mathbf{x}; p]^T$ , the function be  $\mathbf{f} = [\mathbf{h}^T; 0]^T$ , Eq. (6) can be rewritten as

$$\dot{\mathbf{X}} = \mathbf{f}(t, \mathbf{X}, \phi) \quad (8)$$

Assumed that  $\mathbf{X}^* = [\mathbf{X}^*; p^*]^T$  is the nominal state, and the nominal control variable is  $\phi^*(t)$ , the modified dynamical equations based on the nominal trajectory can be got below:

$$\dot{\mathbf{X}}^* = \mathbf{f}(t, \mathbf{X}^*, \phi^*) \quad (9)$$

Supposing the state deviation is expressed as  $\delta\mathbf{X}$ ,  $\delta\phi$  is the control deviation. Given Eq. (8) and Eq. (9), expanding the equation at nominal trajectory so that the deviation caused by the initial state uncertainties or atmosphere model uncertainty factor can be obtained as followed:

$$\delta\dot{\mathbf{X}} = \mathbf{A}(t)\delta\mathbf{X} + \mathbf{B}(t)\delta\phi + \frac{\mathbf{C}(t)}{2}\delta\mathbf{X}^2 \quad (10)$$

Where

$$\left. \frac{\partial \mathbf{f}}{\partial \mathbf{X}} \right|_{\substack{\mathbf{X}(t) = \mathbf{X}^*(t) \\ \phi(t) = \phi^*(t)}} = \mathbf{A}(t)$$

$$\left. \frac{\partial \mathbf{f}}{\partial \phi} \right|_{\substack{\mathbf{X}(t) = \mathbf{X}^*(t) \\ \phi(t) = \phi^*(t)}} = \mathbf{B}(t)$$

$$\left. \frac{\partial^2 \mathbf{f}}{\partial \mathbf{X}^2} \right|_{\substack{\mathbf{X}(t) = \mathbf{X}^*(t) \\ \phi(t) = \phi^*(t)}} = \mathbf{C}(t)$$

So the detailed forms of matrix  $\mathbf{A}_i$  and  $\mathbf{C}_i$  can be got, and denote respectively the  $i$ th row of  $\mathbf{A}(t)$  and  $\mathbf{C}(t)$ . Moreover, the detailed forms are expressed as followed:

$$\begin{aligned} \mathbf{A}_1 &= [0 \quad 0 \quad 0 \quad R \quad vS \quad 0 \quad 0] \\ \mathbf{A}_2 &= \left[ -\frac{G_2}{r} \quad 0 \quad \frac{G_1}{M} \quad G_6 \quad -G_7 \quad G_8 \quad 0 \right] \\ \mathbf{A}_3 &= \left[ -\frac{vH}{r^2} \quad 0 \quad 0 \quad \frac{H}{r} \quad -\frac{vK}{r} \quad -\frac{vE}{r} \quad 0 \right] \\ \mathbf{A}_4 &= \left[ \frac{d}{h_s} + \frac{2gR}{r} \quad 0 \quad 0 \quad -\frac{2d}{v} \quad -gS \quad 0 \quad -\frac{d}{(1+p)} \right] \\ \mathbf{A}_5 &= \left[ -T_1 - L_3S \quad 0 \quad 0 \quad \frac{lW}{v^2} + L_4S \quad L_5R \quad 0 \quad h_sT_2 \right] \\ \mathbf{A}_6 &= \left[ -\frac{G_1}{r} - J_1 \quad 0 \quad \frac{G_2}{M} \quad G_3 + \frac{h_sJ_1}{v} \quad -G_4 + \frac{h_sJ_1R}{S} \quad G_5 \quad h_sJ_2 \right] \\ \mathbf{A}_7 &= [\mathbf{0}_{1 \times 7}] \end{aligned} \quad (11)$$

Where  $E = \cos \gamma \sin \phi$ ,  $F = \sin \gamma \sin \phi$ ,  $H = \cos \gamma \cos \phi$ ,  $K = \sin \gamma \cos \phi$ ,  $M = \cos \lambda$ ,  $N = \sin \lambda$ ,  $W = \cos \phi$ ,  $V = \sin \phi$ ,  $S = \cos \gamma$ ,  $R = \sin \gamma$ ,  $G_1 = \frac{vEN}{rM}$ ,  $G_2 = \frac{vE}{rM}$ ,  $G_3 = \frac{EN}{rM}$ ,  $G_4 = \frac{vFN}{rM}$ ,  $G_5 = \frac{vHN}{rM}$ ,  $G_6 = \frac{E}{rM}$ ,  $G_7 = \frac{vF}{rM}$ ,  $G_8 = \frac{vH}{rM}$ ,  $T_1 = \frac{lW}{h_s v}$ ,  $T_2 = \frac{T_1}{1+p}$ ,  $L_3 = \frac{v}{r^2} - \frac{2g}{rv}$ ,  $L_4 = \frac{g}{v^2} + \frac{1}{r}$ ,  $L_5 = \frac{g}{v} - \frac{v}{r}$ ,  $J_1 = \frac{lV}{h_s vS}$ ,  $J_2 = \frac{J_1}{1+p}$ .

$$\begin{aligned}
 \mathbf{C}_1 &= [\mathbf{0}_{1 \times 25} \quad \mathbf{S} \quad \mathbf{0}_{1 \times 5} \quad \mathbf{S} \quad -v\mathbf{R} \quad \mathbf{0}_{1 \times 16}] \\
 \mathbf{C}_2 &= \begin{bmatrix} \frac{2G_2}{r^2} & 0 & -\frac{G_1}{rM} & -\frac{G_6}{r} & \frac{G_7}{r} & -\frac{G_8}{r} & \mathbf{0}_{1 \times 8} & -\frac{G_1}{rM} & 0 & \frac{vE(1+N^2)}{rM^3} & \frac{G_3}{M} & -\frac{G_4}{M} & \frac{G_5}{M} & 0 \\ -\frac{G_6}{r} & 0 & \frac{G_3}{M} & 0 & -\frac{F}{rM} & \frac{H}{rM} & 0 & \frac{G_7}{r} & 0 & -\frac{G_4}{M} & -\frac{F}{rM} & -G_2 & -\frac{vK}{rM} & 0 & -\frac{G_8}{r} \\ 0 & \frac{G_5}{M} & \frac{H}{rM} & -\frac{vK}{rM} & -G_2 & \mathbf{0}_{1 \times 8} \end{bmatrix} \\
 \mathbf{C}_3 &= \begin{bmatrix} \frac{2vH}{r^3} & \mathbf{0}_{1 \times 2} & -\frac{H}{r^2} & \frac{vK}{r^2} & \frac{vE}{r^2} & \mathbf{0}_{1 \times 15} & -\frac{H}{r^2} & \mathbf{0}_{1 \times 3} & -\frac{K}{r} & -\frac{E}{r} & 0 & \frac{vK}{r^2} & \mathbf{0}_{1 \times 2} \\ -\frac{K}{r} & -\frac{vH}{r} & \frac{vF}{r} & 0 & \frac{vE}{r^2} & \mathbf{0}_{1 \times 2} & -\frac{E}{r} & \frac{vF}{r} & -\frac{vH}{r} & \mathbf{0}_{1 \times 8} \end{bmatrix} \\
 \mathbf{C}_4 &= \begin{bmatrix} -\frac{d}{h_s^2} - \frac{6gR}{r^2} & \mathbf{0}_{1 \times 2} & \frac{2d}{vh_s} & \frac{2gS}{r} & 0 & \frac{d}{(1+p)h_s} & \mathbf{0}_{1 \times 14} & \frac{2d}{vh_s} & \mathbf{0}_{1 \times 2} & -\frac{2d}{v^2} & \mathbf{0}_{1 \times 2} \\ -\frac{2d}{(1+p)v} & \frac{2gS}{r} & \mathbf{0}_{1 \times 3} & gR & \mathbf{0}_{1 \times 9} & \frac{d}{(1+p)h_s} & \mathbf{0}_{1 \times 2} & \frac{2d}{(1+p)v} & \mathbf{0}_{1 \times 3} \end{bmatrix} \\
 \mathbf{C}_5 &= \begin{bmatrix} \frac{T_1}{h_s} - L_1 S & \mathbf{0}_{1 \times 2} & -\frac{T_1}{v} - L_2 S & L_3 R & 0 & -T_2 & \mathbf{0}_{1 \times 14} & -\frac{T_1}{v} - L_2 S & \mathbf{0}_{1 \times 2} & -\frac{2gS}{v^3} & -L_4 R \\ 0 & \frac{T_2 h_s}{v} & L_3 R & \mathbf{0}_{1 \times 2} & -L_4 R & L_5 S & \mathbf{0}_{1 \times 9} & -T_2 & \mathbf{0}_{1 \times 2} & \frac{T_2 h_s}{v} & \mathbf{0}_{1 \times 3} \end{bmatrix} \\
 \mathbf{C}_6 &= \begin{bmatrix} I_1 & 0 & -\frac{G_2}{rM} & I_2 & I_3 & -\frac{G_5}{r} & -J_2 & \mathbf{0}_{1 \times 7} & -\frac{G_2}{rM} & 0 & \frac{2G_1}{M^2} & \frac{G_6}{M} & -\frac{G_7}{M} & \frac{G_8}{M} \\ 0 & I_2 & 0 & \frac{G_6}{M} & 0 & I_4 & G_{10} & \frac{h_s J_1}{v} & I_3 & 0 & -\frac{G_7}{M} & I_4 & J_3 - G_1 & -G_{11} & \frac{J_4}{S} \\ -\frac{G_5}{r} & 0 & \frac{G_8}{M} & G_{10} & -G_{11} & -G_1 & 0 & -J_2 & \mathbf{0}_{1 \times 2} & \frac{h_s J_2}{v} & \frac{J_4}{S} & \mathbf{0}_{1 \times 2} \end{bmatrix} \\
 \mathbf{C}_7 &= [\mathbf{0}_{1 \times 49}]
 \end{aligned} \tag{12}$$

Where  $L_1 = \frac{6g}{r^2 v} - \frac{2v}{r^3}$ ,  $L_2 = \frac{2g}{rv^2} + \frac{1}{r^2}$ ,  $J_3 = \frac{lV(1+R^2)}{rS^3}$ ,  $J_4 = \frac{lVR}{v(1+p)S}$ ,  $G_9 = \frac{FN}{rM}$ ,  $G_{10} = \frac{HN}{rM}$ ,  $G_{11} = \frac{vKN}{rM}$ ,  $I_1 = \frac{2G_1}{r^2} + \frac{J_1}{h_s}$ ,  $I_2 = -\frac{G_3}{r} - \frac{J_1}{v}$ ,  $I_3 = \frac{G_4}{r} - \frac{J_1 R}{S}$ ,  $I_4 = \frac{lVR}{v^2 S^2} - G_9$ .

Now the uncertainty propagation, based on initial condition and atmosphere model uncertainty, can be researched. However, the control uncertainty is not discussed here.

### 3 Simulation and results

In this section, the effect of the uncertainties of initial conditions and atmosphere model uncertainty on spacecraft flight trajectories in the Mars entry phase is researched based on IM method, and the results are compared with those

obtained by LL method and MC method.

In order to discuss the laws of uncertainty propagation and the range of application for IM method, a series of initial errors are assumed to each system state or atmosphere model uncertainty to sum up the laws of uncertainty propagation for dynamic system of Mars entry. The nominal initial state variables and physical parameters of entry spacecraft are listed in Table 1, and the consequences based on each of these cases are plotted in Fig. 1—Fig. 7.

**Table 1** Nominal initial state variables and physical parameters

| Symbol                 | Quantity              | Value |
|------------------------|-----------------------|-------|
| $h_0/\text{km}$        | Initial altitude      | 125   |
| $\theta_0/(^{\circ})$  | Initial longitude     | 0     |
| $\lambda_0/(^{\circ})$ | Initial latitude      | 1     |
| $v_0/(\text{m/s})$     | Initial velocity      | 6900  |
| $\gamma_0/(^{\circ})$  | Initial FPA           | -13   |
| $\phi_0/(^{\circ})$    | Initial azimuth angle | 89    |
| $L/D$                  | Lift-to-drag ratio    | 0.156 |
| $B/(\text{kg/m}^2)$    | Ballistic coefficient | 62.5  |

Figure 1 shows that when the initial state uncertainty of radius increases from  $-4\text{ km}$  to  $4\text{ km}$ , the other initial state uncertainties remain zero. The final uncertainties of longitude and latitude are in close proximity to linearity. But

the errors of the other states change quickly. So they go out of the linear domain quickly. The reason is that there exists perturbations from initial nominal radius which brings in corresponding errors to all the other states. The first column of matrix **A** and the first to the seventh columns of matrix **C** are very small, with the result that the errors introduced by  $r$  are also small. However, when the error of  $\gamma$  enhances with the increasing of radius perturbation, it affects  $v$ ,  $r$  and  $\phi$  heavily in turn. Because the corresponding items of **A** and matrix **C** are relatively large. Furthermore, IM method is better than LL method.

Figure 2 indicates that the final longitude error keeps the same with the initial state uncertainty error. That's because the corresponding items of matrix **A** and matrix **C** are zero. The perturbations about  $\theta$  do not influence the other

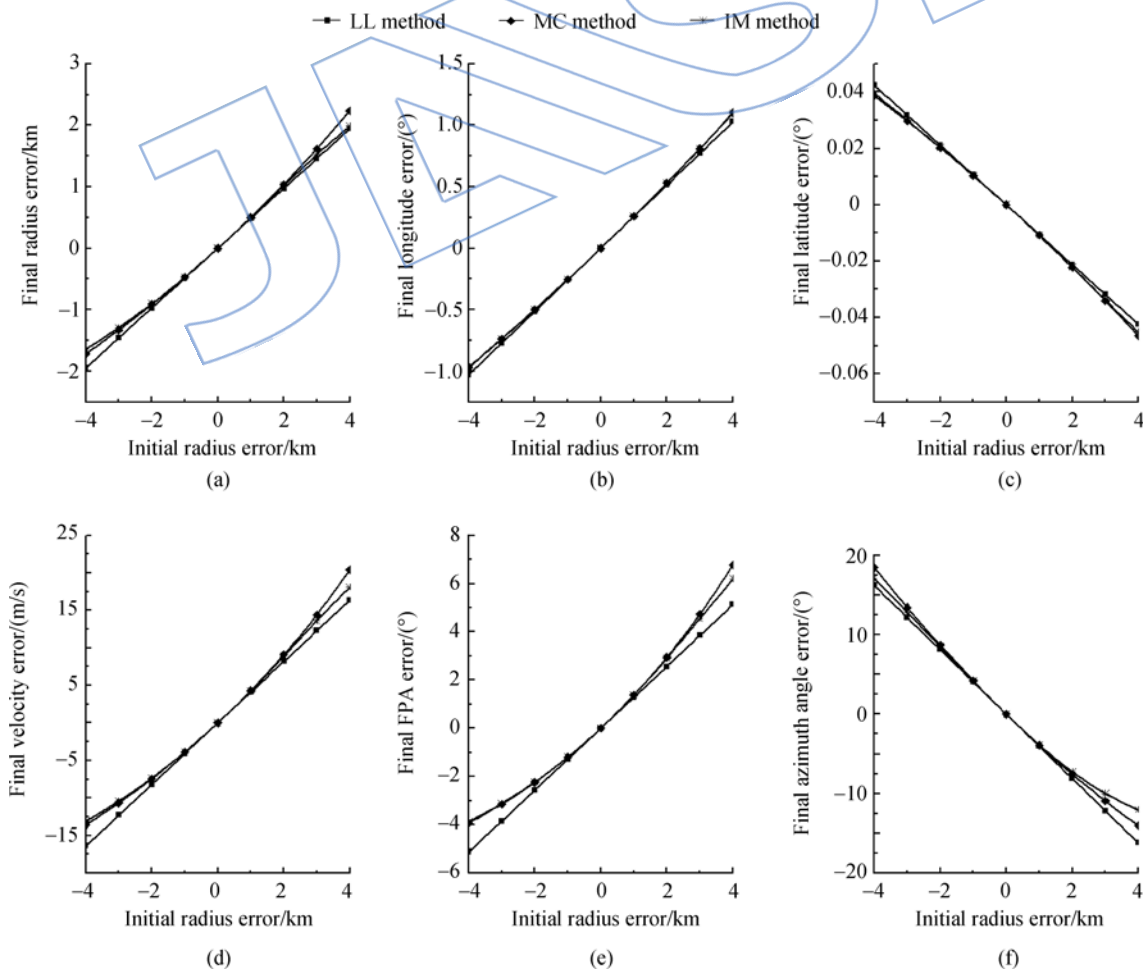


Fig. 1 Final errors caused by initial radius error

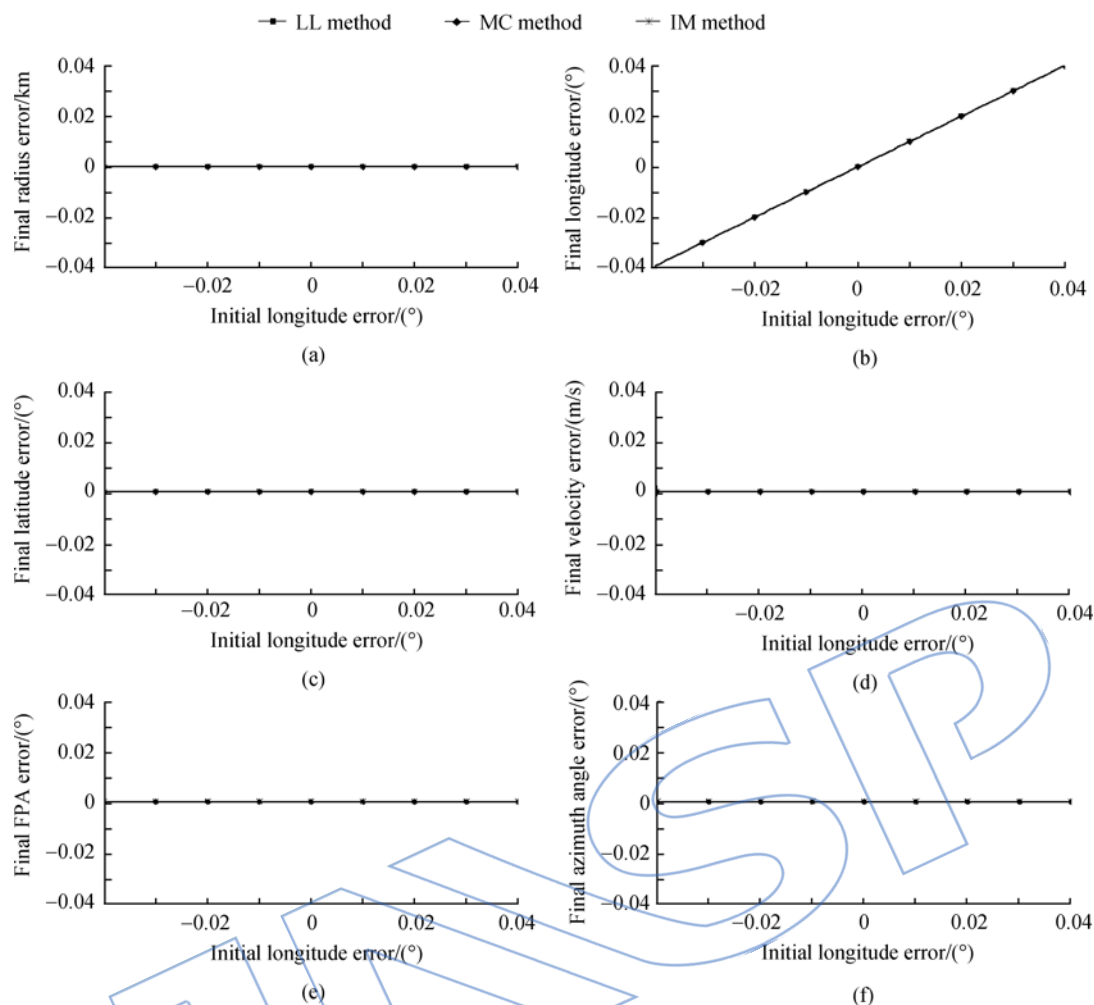


Fig. 2 Final errors caused by initial longitude error

states, so the other states remain the same and do not have an effect on  $\theta$  in turn.

Figure 3 shows that the uncertainty of latitude only affects  $\theta$  and  $\psi$ . The reason is that only the third columns of matrix  $\mathbf{A}_2$  and matrix  $\mathbf{A}_6$ , and the fifteenth to the twenty-first columns of matrix  $\mathbf{C}_2$  and matrix  $\mathbf{C}_6$  have the error introduced by  $\lambda$ . However, the errors are all small. As a result of that, the effect of  $\lambda$  on  $\theta$  and  $\psi$  is little. Moreover, IM method is also better than LL method.

The initial uncertainty of velocity increases from  $-20$  m/s to  $20$  m/s, and the result is that the errors of all states are in close proximity to linearity. It is found that the items by studying the fourth column of  $\mathbf{A}$  and the twenty-second to twenty-eighth columns of  $\mathbf{C}$  are all small. Moreover, that's why the linear domain is relatively large. And the Fig. 4 indicates the case.

Figure 5 shows that the corresponding items of  $\mathbf{A}$  and  $\mathbf{C}$  are relatively large relative to the other items. This explains why the uncertainty of FPA affects  $r$  and  $v$  heavily. Moreover,  $r$  and  $v$  make an impact on each other relatively. So when the initial uncertainty  $\gamma$  increases from  $-0.2^\circ$  to  $0.2^\circ$ , the errors of  $r$  and  $v$  have strong variation and go beyond the linearity range quickly. From Fig. 5, it shows that the simulation of LL method is worse than IM method.

Figure 6 indicates that the perturbations of the azimuth angle only have an effect on  $\theta$ ,  $\lambda$  and itself. But  $\theta$  does not affect the other states. And  $\lambda$  only makes an impact on  $\theta$  and  $\psi$ . So the perturbations of the azimuth angle do not influence  $r$ ,  $v$  and  $\gamma$ .

The atmosphere model uncertainty affects  $v$  and  $\gamma$  heavily. The reason is that the atmosphere model uncertainty affects  $\gamma$  heavily, so  $\gamma$  has an

effect on  $r$  and  $v$ . Moreover, they influence each other interactively. What is more, the IM method is almost close to the MC method from Fig. 7.

From Fig. 1 to Fig. 7, if the initial radius uncertainty is between  $-2$  km and  $2$  km, the initial

longitude uncertainty ranges from  $-0.04^\circ$  to  $0.04^\circ$ ; the latitude uncertainty is between  $-0.04^\circ$  and  $0.04^\circ$ ; the scope of velocity uncertainty is from  $-20$  m/s to  $20$  m/s; the flight path angle should ranges from  $-0.1^\circ$  to  $0.1^\circ$ ; between  $-1^\circ$  and  $1^\circ$

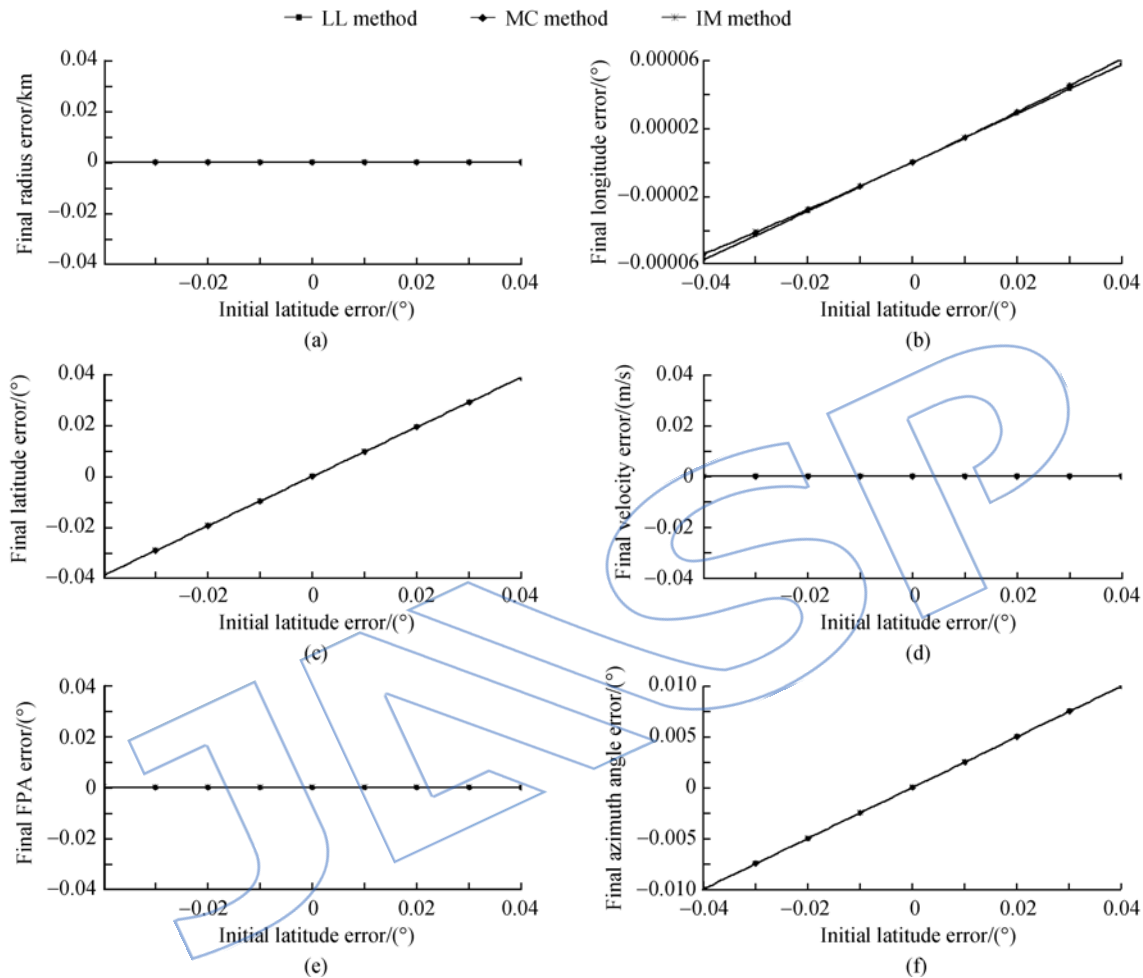
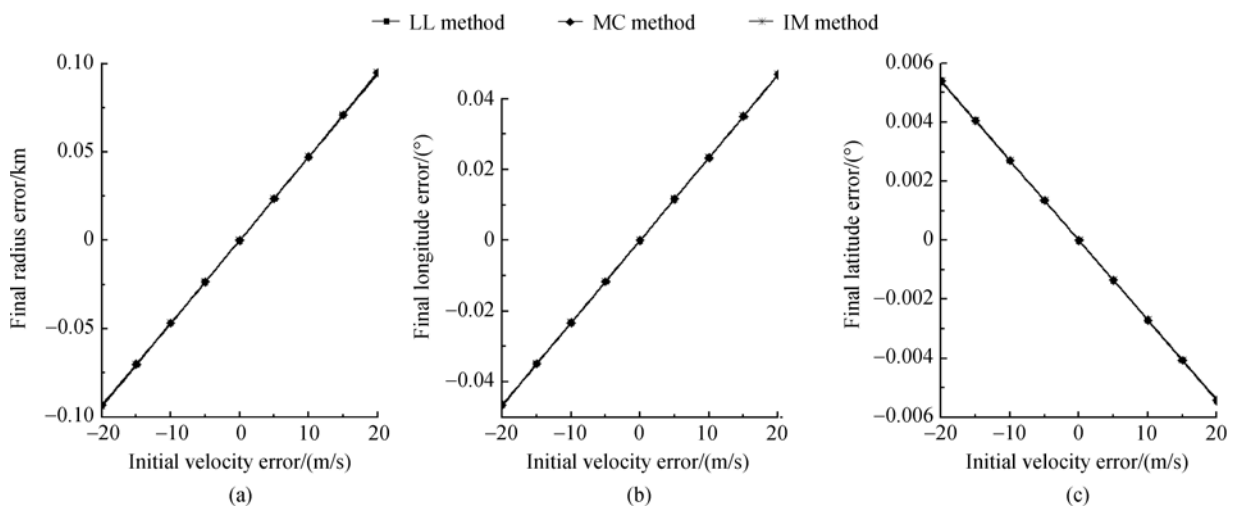


Fig. 3 Final errors caused by initial latitude error



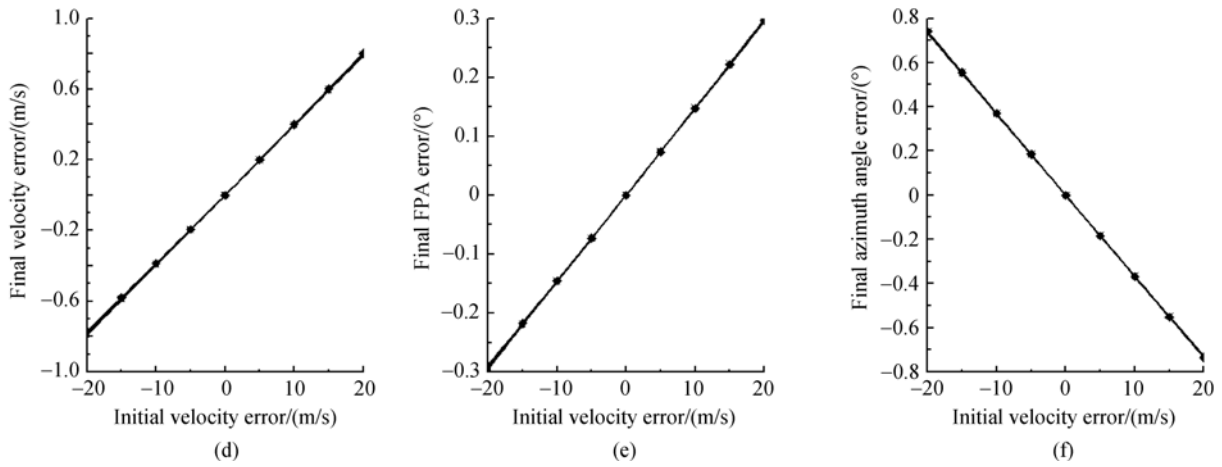


Fig. 4 Final errors caused by initial velocity error

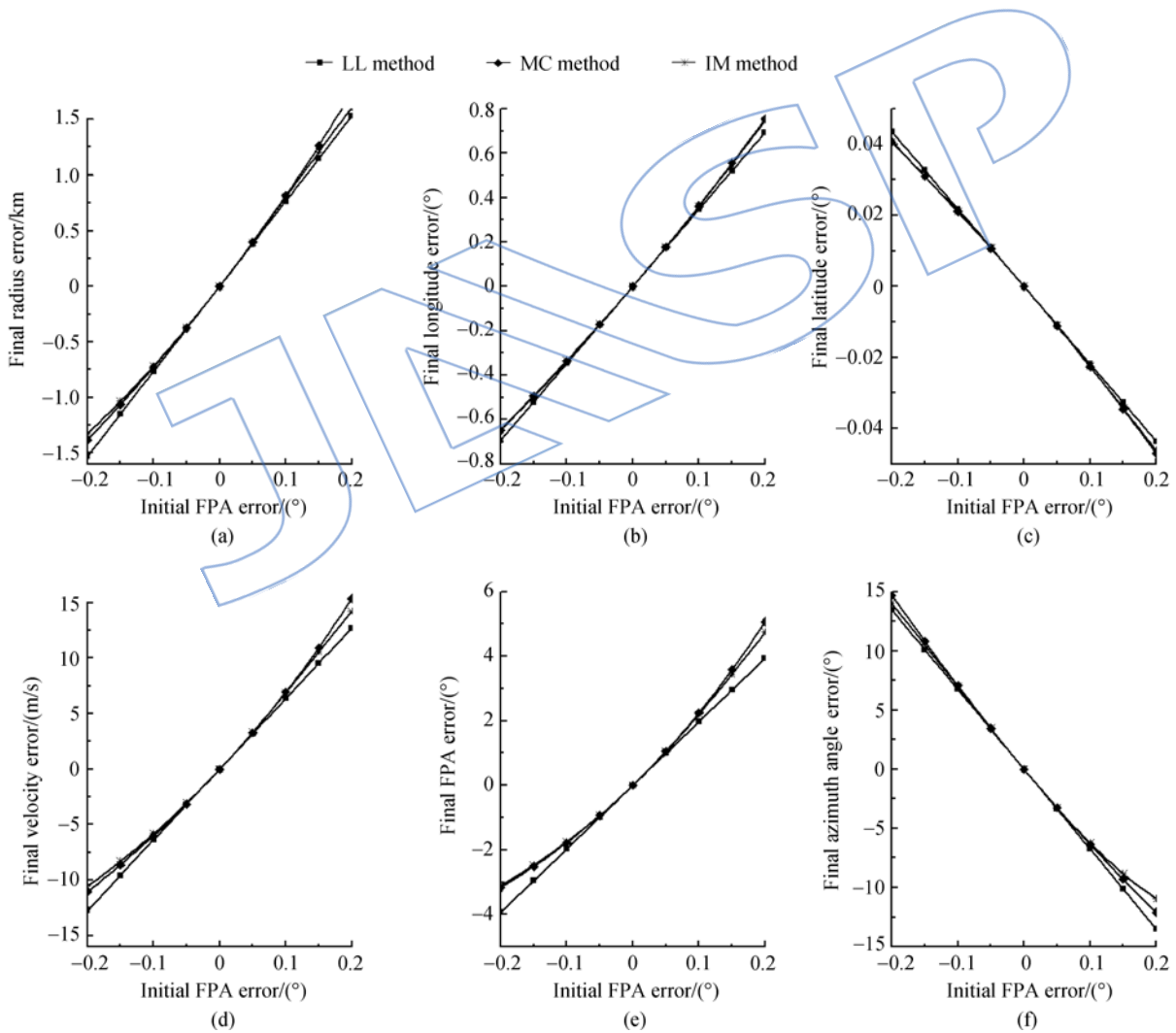


Fig. 5 Final errors caused by initial FPA error

should be the scope of azimuth angle uncertainty, and from  $-15\%$  to  $15\%$  should be the domain of atmosphere model uncertainty; the simulation results from IM method and MC method are almost

the same.

Given the analysis above, an analysis associated with all the initial uncertainties is performed. The IM method presented in this paper

gets the results so quickly, however, MC method (10 000 times) spends more than one hour getting the results. And Fig. 8 shows that the

simulation of IM method is almost the same to the MC method during the Mars entry phase.

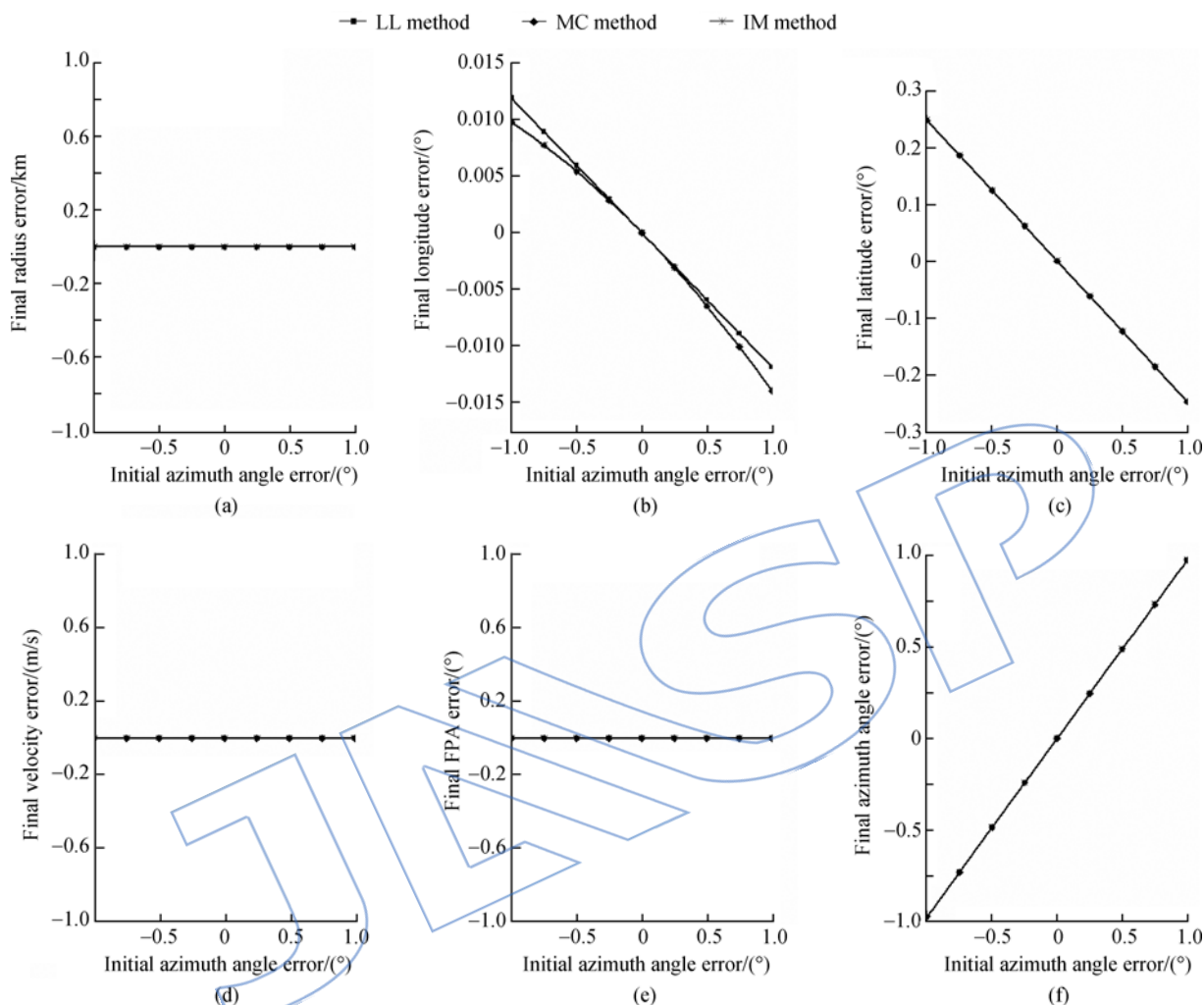
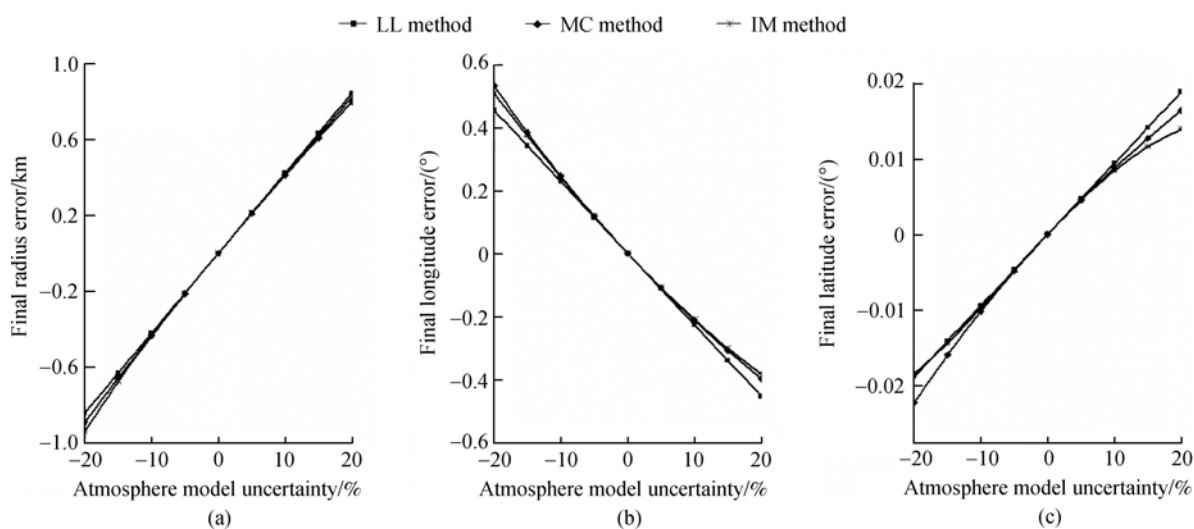


Fig. 6 Final errors caused by initial azimuth error



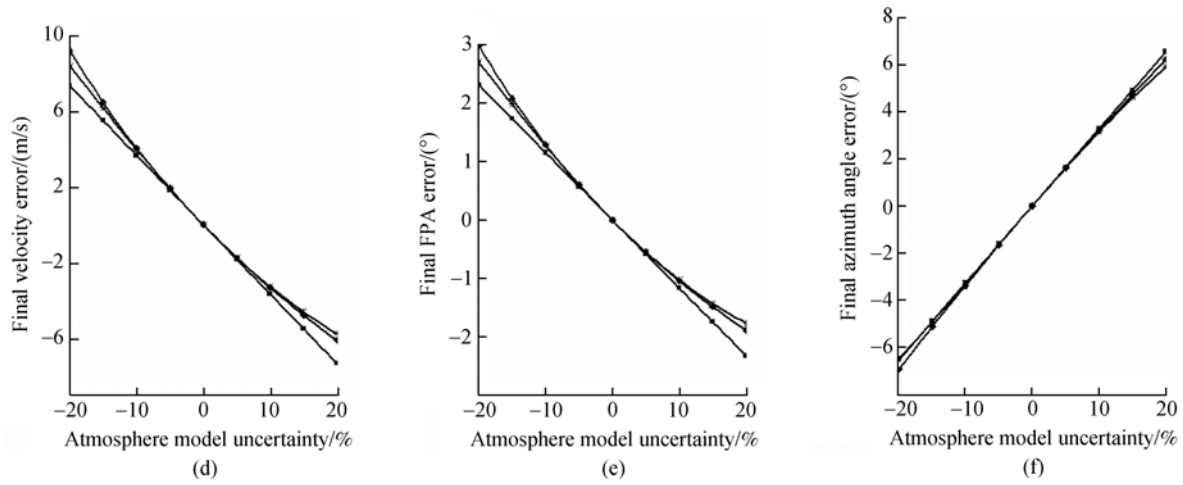


Fig. 7 Final errors caused by initial atmosphere model uncertainty

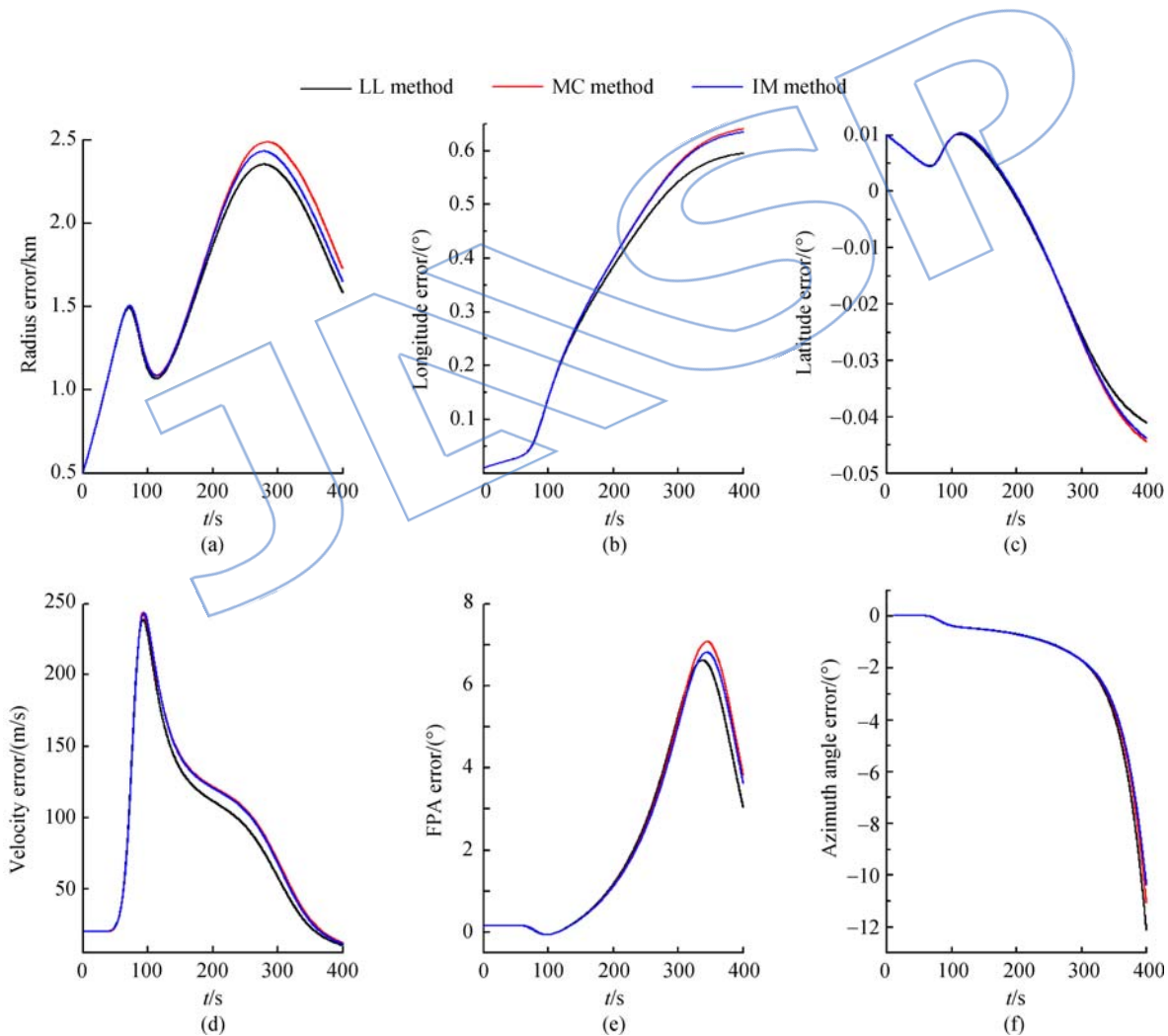


Fig. 8 Comparison of results of IM method with LL method and MC method

#### 4 Conclusion

During the Mars entry phase, the uncertainty propagation laws of dynamic model are

analyzed by using the IM method. Then the range of application for future research is also studied through the results of the IM method, LL method and MC method. Furthermore, an

analysis associated with all the initial uncertainties is performed. From Fig. 1 to Fig. 8, the conclusions can be drawn as followed:

1) The initial errors of radius, velocity, FPA and atmosphere model uncertainty are momentous factors which have an effect on all the states and the application of IM method, and the vital reasons for affecting the states are the two factors which are FPA error and atmosphere model uncertainty.

2) The perturbation of longitude makes no impact on the other states. As a result of that, the final errors of the other states are all zero, and the final longitude error keeps the same to the initial longitude error.

3) The azimuth angle error and the latitude error only affect the state of longitude and themselves. Moreover, the errors of azimuth angle, longitude and latitude are almost changing in a linear fashion.

4) The IM method simulation shows that the IM method is better than the LL method, and the IM method keeps practically the same to the MC method during the Mars entry phase.

## References:

- [1] Braun R D. Mars exploration entry, descent, and landing challenges[J]. *Journal of Spacecraft and Rockets*, 2007, 44 (2): 310-323.
- [2] Brand T, Fuhrman L, Geller D, et al. GN&C technology needed to achieve pinpoint landing accuracy at Mars[R]. AIAA-2004-4748, 2004.
- [3] Desai P N, Schoenenberger M, Cheatwood F M. Mars exploration rover six-degree-of-freedom entry trajectory analysis[J]. *Journal of Spacecraft and Rockets*, 2006, 43 (5): 1019-1025.
- [4] Wolf A A, Graves C, Powell R, et al. Systems for pinpoint landing at Mars[R]. Maui, US: AAS/AIAA 14th Space Flight Mechanics Meeting, 2005.
- [5] LI Shuang, ZHANG Liu. Autonomous navigation and guidance scheme for precise and safe planetary landing[C]. *Aircraft Engineering and Aerospace Technology*, 2009, 81 (6): 516-521.
- [6] Chu C C. Development of advanced entry, descent, and landing technologies for future Mars missions[C] // *Proceedings of IEEE Aerospace Conference*. Big Sky, US: IEEE, 2006: 4-11.
- [7] Way D W, Powell R W, Chen A, et al. Mars Science Laboratory: entry, descent, and landing system performance[C] // *Proceedings of the 2007 IEEE Aerospace Conference*. Big Sky, US: IEEE, 2007: 1-19.
- [8] SHEN Haijun, Seywald H, Powell R W. Desensitizing the pin-point landing trajectory on Mars [R]. AIAA-2008-6943, 2008.
- [9] Caflisch R E. Monte Carlo and quasi-Monte Carlo methods [J]. *Acta Numerica*, 1998, 7: 1-49.
- [10] Wolfhard J. Rugged free energy landscapes: common computational approaches to spin glasses, structural glasses and biological macromolecules [M]. Berlin, Germany: Springer-Verlag, 2008.
- [11] Brown R G, Hwang P Y C. Introduction to random signals and applied Kalman filtering [M]. Chichester, UK: Wiley, 1992.
- [12] REN Gaofeng, CUI Hutao, CUI Pingyuan, et al. A rapid uncertainty propagation method for pre-parachute phase of Mars entry[C] // *Proceedings of the 2011 Chinese Control and Decision Conference*. Mianyang Sichuan: IEEE, 2011: 3124-3147. (in Chinese)
- [13] Julier S J, Uhlmann J K. Unscented filtering and nonlinear estimation[J]. *Proceedings of the IEEE*, 2004, 92 (3): 401-422.
- [14] Sengupta P, Bhattacharya R. Uncertainty analysis of hypersonic flight using multi-resolution Markov operators [R]. AIAA-2008-6298, 2008.
- [15] Prabhakar A, Fisher J, Bhattacharya R. Polynomial chaos-based analysis of probabilistic uncertainty in hypersonic flight dynamics[J]. *Journal of Guidance, Control, and Dynamics*, 2010, 33(1): 222-234.
- [16] Park S H. Nonlinear trajectory[D]. Michigan: University of Michigan, 2007.